**Lecture 13 Spatio-Temporal Models**

**13.1 The joint covariance**

“Of even greater interest and need than temporal or spatial models and methods separately for environmental studies are models and methods which conjointly encompass the effects of both time and space. This is a very difficult area of work, and progress on truly conjoint spatial-temporal models and methods is limited” (Barnett, 2004).

The spatio-temporal random process is denoted by {*Z*(***s***, *t*)}, indexed by special coordinate ***s*** and time coordinate *t*. In most of applications, *Z* is univariate, the spatial coordinate is continuous and two-dimensional, and the time coordinate is discrete. The general spatio-temporal model may be written as

*Z*(***s***, *t*) = *W*(***s***, *t*) + ɛ(***s***, *t*),

where *W*(***s***, *t*) is a basic data model and ɛ(***s***, *t*) is the residual term. Further assumptions are made on *W*(***s***, *t*) in various approaches and methods. For example,

*W*(***s***, *t*) = *µ*(***s***, *t*) + υ(***s***, *t*),

where *µ*(***s***, *t*) is a deterministic function of ***s*** and *t*, and υ(***s***, *t*) is a further error term attached to *µ*(***s***, *t*).

The ***spatio-temporal covariance function*** is defined as,

C(,;,) = Cov[Z(,), Z(,)] .

The process is said to be ***covariance stationary*** if the expectation of *Z*(***s***, *t*) is independent of ***s*** and *t* and if

C(,;,) = C(-;-) = C(*h*; *k*),

where *h* is the distance between location  and location  and *k* is the time lag between time  and time .

Note that the stationarity assumption is a strong constraint and may not be always justified in applications.

The simplest situation in spatio-temporal modelling is one where the process {*Z*(***s***, *t*)} is stationary and also separable in the sense that

C(*h*; *k*) = C1(*h*)C2(*k*)

or C(*h*; *k*) = C1(*h*) + C2(*k*),

which can lead to distinct handling of the spatial and temporal components. More often, however, we need to deal with non-separable processes.

**13.2 Example**

Paper attached: *Introduction to Spatio-Temporal Variography*, by E. Pebesma

**Reference**

Barnett, V., (2004), *Environmental Statistics: Methods and Applications*, Wiley.